The comparison method of the magnetometers data with different orientation of their sensors.

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Abstract

Features of two basic methods of checking the long-term stability and own noise level of three-component magnetometers are considered. The mathematical model of one of them is proposed. The magnetometers data processing based on the linear regression method is discussed.

Introduction

The measurement of long period (from minutes to years) variations of the Earth's magnetic field has particular importance for solving of some important global geophysical problems. The high sensitivity three-component magnetometers are widely used for this purpose. Taking into account low level and specific frequency range of measured signals, one may say that long-term stability and noise level are the most important metrological features of these magnetometers. It is hard work to define these features, because it is necessary to separate own magnetometer signals and magnetic disturbances both of Earth's field and of field of artificial sources. Two techniques of solving this task can be proposed:

1. Magnetic shielding method – magnetometer sensor is allocated into shielded space, which is free from the outer magnetic field influence.
2. Differential method – the measurements carried out by two magnetometers: reference and tested. The tested magnetometer data quality estimation is realized by subtracting of the both magnetometer data.

First method has the following disadvantage. Magnetic field into the shield is not zero. The residual field in the material, from which the shield is manufactured, is exposed to the thermal and temporal factors influence with pronounced hysteresis. Because of this noise level and zero drift measurements are possible during limited time interval only, when the influence of thermal and temporal factors is negligible. Another limitation of the method is the impossibility to create the homogeneous and stable magnetic field into the shield with magnitude close to Earth's one. Therefore, it is impossible to check multiplicative error of the magnetometer and specific errors caused by the sensor's axis rotation in the orthogonal field. Nevertheless, this method is widely used for the sensor noise level measurement in frequency range from thousandth Hertz and higher. Also it could be used for metrological features checking of the electronic units.

The second method is irreplaceable for definition of noise level and zero drift during more long time interval - down to month and more. The suppression of outer signals or the method tolerance directly depends on parameters characterizing the single sensor and the ratio between sensors. As a rule, these parameters are unknown *a priori*. The installation of corresponding components of the sensors mutually coaxial with high precision constitutes particular difficulties. First of all the existing nonorthogonality of components does not allow to reach the high accuracy. Thus, the determination of magnetometer long-term stability is connected with the measurement of the whole set of metrological features: scale factors, relative angles between sensor components etc. The high accuracy orthogonality and sensitivity tests are realized only at selected observatory, for example in Nurmijarvi (Finland). High-quality three-component coil system with reference magnetometer allows the measuring of sensors sensitivity and orthogonality with errors 0.01% and 1’ accordingly. However, using the possibility of such observatory has no economical reason for preliminary or trial tests. Thus, the development of new methods solving the problem under usual observatory conditions is still actual.

The mathematical model of the differential method

Let us formulate the problem in mathematical terms. Let magnetometer sensors are in homogeneous magnetic field. Output of any component is equal to the sum of external magnetic field along its sensitivity axis and own additive error:

$$U=(B+D)\cdot k, \tag{1}$$

where U – output signal;
B – magnetic field;
D - additive error;
k – scale factor.

Let components are marked by symbols X,Y,Z. The subscripts r and t will indicate the belonging of some parameters to the reference and test magnetometer accordingly. Then, output signals of each component are:
for reference magnetometer

$$\begin{aligned} Xr &= (B_{xr} + D_{xr})\cdot K_{xr} \\ Y_r &= (B_{yr} + D_{yr})\cdot K_{yr} \\ Z_r &= (B_{zr} + D_{zr})\cdot K_{zr} \end{aligned} \tag{2}$$

for test magnetometer

$$\begin{aligned} X_t &= (B_{xt} + D_{xt})\cdot K_{xt} \\ Y_t &= (B_{yt} + D_{yt})\cdot K_{yt} \\ Z_t &= (B_{zt} + D_{zt})\cdot K_{zt} \end{aligned} \tag{3}$$

Let us direct the axies of Cartesian coordinate system in the way as shown in Fig. 1. Reference magnetometer axes X_r, Y_r are in horizontal plane *x,y* and the Y_r axis is pointing in parallel to the *y* axis. Axes X_r and *x* form angle β. Axes Z_r and *z* form angle γ. The horizontal plane projection of Z_r axis and *y* axis form angle α.

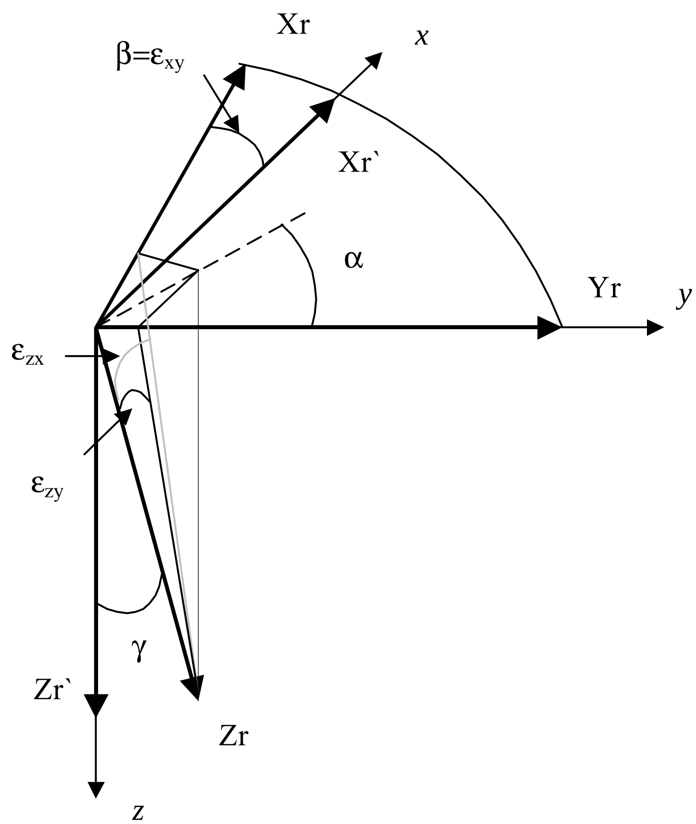

Fig. 1. The reference magnetometer axes position in selected coordinate system.

The nonorthogonality angles of the reference magnetometer can be expressed through α,β,γ by formulas:

$$\begin{aligned} &\varepsilon_{xyr}=\beta \\ &\varepsilon_{yzr}=-\arcsin[\sin(\gamma)\cdot\sin(\alpha)] \\ &\varepsilon_{xzr}=\arcsin[\sin(\gamma)\cdot\sin(\beta-\alpha)] \end{aligned} \qquad (4)$$

It is possible to express values of magnetic field in the directions of selected coordinate system axes:

$$\begin{aligned} &B_{xr}` = B_{xr}\cdot[\cos(\beta)]^{-1} + B_{yr}\cdot tg(\beta) \\ &B_{yr}` = B_{yr} \\ &B_{zr}` = - B_{xr}\cdot[\cos(\beta)]^{-1}\cdot\sin(\alpha)\cdot tg(\gamma) - B_{yr}\cdot[tg(\beta)\cdot\sin(\alpha) + \cos(\alpha)]\cdot tg(\gamma) + B_{zr}\cdot[\cos(\gamma)]^{-1} \end{aligned} \qquad (5)$$

Directions of test magnetometer components are defined by three pairs of angles - φ_x, θ_x; φ_y, θ_y; φ_z, θ_z (fig. 2).

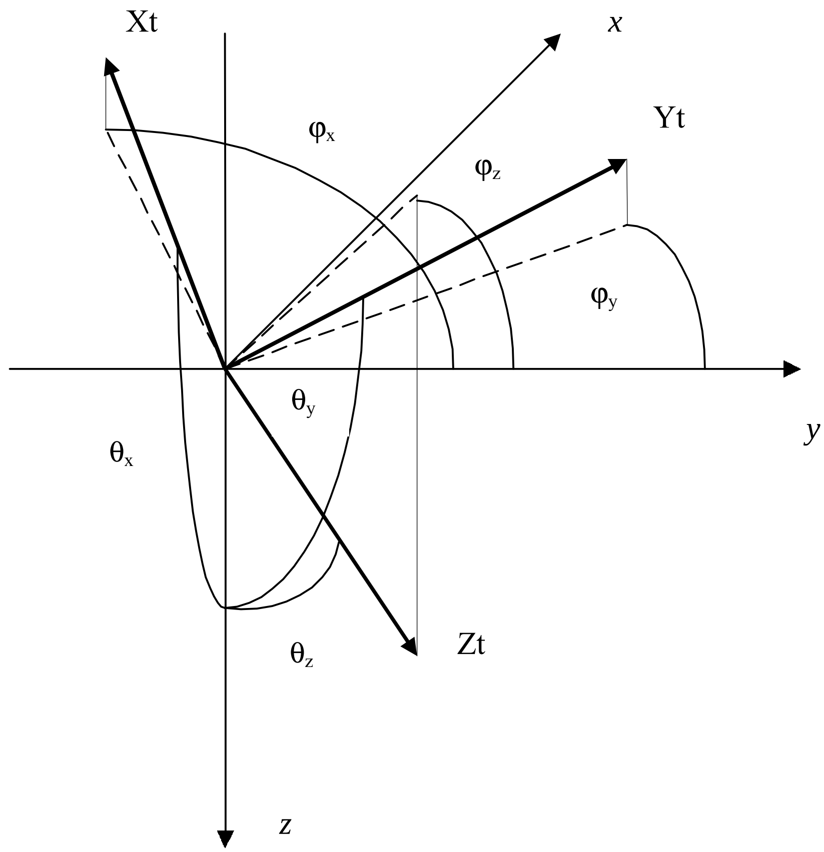

Fig. 2. The test magnetometer axes position in selected coordinate system.

Magnetic field in the directions of test magnetometer axes are expressed by formulas:

$$
\begin{aligned}
B_{xt} &= B_{xr}^{`}\cdot\sin(\varphi_x)\cdot\sin(\theta_x) + B_{yr}^{`}\cdot\cos(\varphi_x)\cdot\sin(\theta_x) + B_{zr}^{`}\cdot\cos(\theta_x) \\
B_{yt} &= B_{xr}^{`}\cdot\sin(\varphi_y)\cdot\sin(\theta_y) + B_{yr}^{`}\cdot\cos(\varphi y)\cdot\sin(\theta_y) + B_{zr}^{`}\cdot\cos(\theta_y) \qquad (6) \\
B_{zt} &= B_{xr}^{`}\cdot\sin(\varphi_z)\cdot\sin(\theta_z) + B_{yr}^{`}\cdot\cos(\varphi_z)\cdot\sin(\theta_z) + B_{zr}^{`}\cdot\cos(\theta_z)
\end{aligned}
$$

After substituting (5) to (6) and taking designations:

$$
\begin{aligned}
B_{xt} &= B_{xr}\cdot[\sin(\varphi_x)\cdot\sin(\theta_x) - \sin(\alpha)\cdot tg(\gamma)\cdot\cos(\theta_x)]\cdot[\cos(\beta)]^{-1} + \\
&+ B_y\cdot[\cos(\varphi_x)\cdot\sin(\theta_x) + tg(\beta)\cdot\sin(\varphi_x)\cdot \sin(\theta_x) - (tg(\beta)\cdot\sin(\alpha) + \cos(\alpha))\cdot tg(\gamma)\cdot\cos(\theta_x)] + \\
&+ B_z\cdot\cos(\theta_x)\cdot [\cos(\gamma)]^{-1} \\
B_{yt} &= B_{xr}\cdot[\sin(\varphi_y)\cdot\sin(\theta_y) - \sin(\alpha)\cdot tg(\gamma)\cdot\cos(\theta_y)]\cdot[\cos(\beta)]^{-1} + \\
&+ B_y\cdot[\cos(\varphi_y)\cdot\sin(\theta_y) + tg(\beta)\cdot\sin(\varphi_y)\cdot \sin(\theta_y) - (tg(\beta)\cdot\sin(\alpha) + \cos(\alpha))\cdot tg(\gamma)\cdot\cos(\theta_y)] + \\
&+ B_z\cdot\cos(\theta_y)\cdot [\cos(\gamma)]^{-1} \\
B_{zt} &= B_{xr}\cdot[\sin(\varphi_z)\cdot\sin(\theta_z) - \sin(\alpha)\cdot tg(\gamma)\cdot\cos(\theta_z)]\cdot[\cos(\beta)]^{-1} + \\
&+ B_y\cdot[\cos(\varphi_z)\cdot\sin(\theta_z) + tg(\beta)\cdot\sin(\varphi_z)\cdot \sin(\theta_z) - (tg(\beta)\cdot\sin(\alpha) + \cos(\alpha))\cdot tg(\gamma)\cdot\cos(\theta_z)] + \\
&+ B_z\cdot\cos(\theta_z)\cdot [\cos(\gamma)]^{-1}
\end{aligned}
\tag{7}
$$

it is posible to write it in more compact way, designating trigonometrical expressions by correspondig elements of the matrix **m**:

$$
\begin{aligned}
B_{xt} &= B_{xr}\cdot m_{11} + B_y\cdot m_{21} + B_z\cdot m_{31} \\
B_{yt} &= B_{xr}\cdot m_{12} + B_y\cdot m_{22} + B_z\cdot m_{32} \\
B_{zt} &= B_{xr}\cdot m_{13} + B_y\cdot m_{23} + B_z\cdot m_{33}
\end{aligned}
\tag{8}
$$

Values of the magnetic field can be expressed in magnetometer readings and its additive errors:

$$
\begin{aligned}
X_t &= X_r\cdot(K_{xt}/K_{xr})\cdot m_{11} + Y_r\cdot(K_{xt}/K_{yr})\cdot m_{21} + Z_r\cdot(K_{xt}/K_{zr})\cdot m_{31} + \\
&+ K_{xt}\cdot[D_{xt} - D_{xr}\cdot m_{11} - D_{yr}\cdot m_{21} - D_{zr}\cdot m_{31}] \\
Y_t &= X_r\cdot(K_{yt}/K_{xr})\cdot m_{12} + Y_r\cdot(K_{yt}/K_{yr})\cdot m_{22} + Z_r\cdot(K_{yt}/K_{zr})\cdot m_{32} + \\
&+ K_{yt}\cdot[D_{yt} - D_{xr}\cdot m_{12} - D_{yr}\cdot m_{22} - D_{zr}\cdot m_{32}] \\
Z_t &= X_r\cdot(K_{zt}/K_{xr})\cdot m11 + Y_r\cdot(K_{zt}/K_{yr})\cdot m_{23} + Z_r\cdot(K_{zt}/K_{zr})\cdot m_{33} + \\
&+ K_{zt}\cdot[D_{zt} - D_{xr}\cdot m_{13} - D_{yr}\cdot m_{23} - D_{zr}\cdot m_{33}]
\end{aligned}
\tag{9}
$$

Let us introduce five parameters, which indicate ratio between scale factors of magnetometers:

$$
p = K_{yt}/ K_{yr} ,q_{rxy} = K_{xr}/ K_{yr} ,\ q_{rzy} = K_{zr}/ K_{yr} ,\ q_{txy} = K_{xt}/ K_{yt} ,\ q_{txy} = K_{xt}/ K_{yt} \tag{10}
$$

Then, the transformation matrix, which connect readings of reference and test magnetometers, is defined as:

$$\mathbf{n} = \begin{pmatrix} (q_{txy}/q_{rxy})\cdot m_{11} & (1/q_{rxy})\cdot m_{12} & (q_{tzy}/q_{rxy})\cdot m_{13} \\ q_{txy}\cdot m_{21} & m_{22} & q_{tzy}\cdot m_{23} \\ (q_{txy}/q_{rzy})\cdot m_{31} & (1/q_{rzy})\cdot m_{32} & (q_{tzy}/q_{rzy})\cdot m_{33} \end{pmatrix}\cdot p \qquad (11)$$

Let us also denote differenial errors as:

$$\begin{aligned} \Delta_x &= K_{xt}\cdot[D_{xt} - D_{xr}\cdot m_{11} - D_{yr}\cdot m_{21} - D_{zr}\cdot m_{31}] \\ \Delta_y &= K_{yt}\cdot[D_{yt} - D_{xr}\cdot m_{12} - D_{yr}\cdot m_{22} - D_{zr}\cdot m_{32}] \\ \Delta_z &= K_{zt}\cdot[D_{zt} - D_{xr}\cdot m_{13} - D_{yr}\cdot m_{23} - D_{zr}\cdot m_{33}] \end{aligned} \qquad (12)$$

After substituting (11),(12) in (9):

$$\begin{aligned} X_t &= X_r\cdot n_{11} + Y_r\cdot n_{21} + Z_r\cdot n_{31} + \Delta_x \\ Y_t &= X_r\cdot n_{12} + Y_r\cdot n_{22} + Z_r\cdot n_{32} + \Delta_y \\ Z_t &= X_r\cdot n_{13} + Y_r\cdot n_{23} + Z_r\cdot n_{33} + \Delta_z \end{aligned} \qquad (13)$$

Let us write it as a matrix equation:

$$\mathbf{T}=\mathbf{R}\cdot\mathbf{n}+\mathbf{d} \qquad (14)$$

where **T**, **R** – N×3 matrix of readings of reference and test magnetometers;
d – the same size matrix of errors;
N – the number of readings.

The linear regression method

So, it is necessary to find values of the transform matrix **n** and the matrix of errors **d**, using readings of reference and test magnetometers. For equations like this the solution algoritm depends on *a priori* information about features of a measuring process. Let us examine conditions of the test. First, Earth’s magnetic field and magnetometers errors have different physical sources. Then, one may say, that these signals, on the whole, are independent. However, the correlation coefficient between some realizations of these processes are may be not equal to zero. For example, it’s possible if both of signals have a temporal drift according to the linear low. In our case it is quite realistic to accept that additive errors of each component is caused by the superposition of the two processes:

a) ‘flicker’ noise with normal distribution;
b) temporal drift under the exponential low.

Second, as a rule the flux-gate instruments for geomagnetic measurements are designed as variometers. Hence, a residual between outer field and own compensator field is measured. A compensator full-scale error causes great systematic part of the additive error.

In the third place, Earth's magnetic field vector changes both in the amplitude and in the direction. It means that signals in the directions of sensitivity axes of a three-component magnetometer have no linear dependence . Taking into account aforesaid things, it is proposed to solve this problem in two stage. At first, it's necessary to find decision for the next model:

$$\begin{aligned} X_t - \overline{X}_t &= \left(X_r - \overline{X}_r\right)\cdot n_{11} + \left(Y_r - \overline{Y}_r\right)\cdot n_{21} + \left(Z_r - \overline{Z}_r\right)\cdot n_{31} + (\Delta_x - \overline{\Delta}_x) \\ Y_t - \overline{Y}_t &= \left(X_r - \overline{X}_r\right)\cdot n_{21} + \left(Y_r - \overline{Y}_r\right)\cdot n_{22} + \left(Z_r - \overline{Z}_r\right)\cdot n_{32} + (\Delta_y - \overline{\Delta}_y) \\ Z_t - \overline{Z}_t &= \left(X_r - \overline{X}_r\right)\cdot n_{31} + \left(Y_r - \overline{Y}_r\right)\cdot n_{33} + \left(Z_r - \overline{Z}_r\right)\cdot n_{33} + (\Delta_z - \overline{\Delta}_z) \end{aligned} \quad (15),$$

or in the matrix form:

$$\mathbf{T_0} = \mathbf{R_0}\cdot\mathbf{n} - \mathbf{d_0} \quad (16)$$

The method of the multiply linear regression [1] gives the following estimation of the transformartion martix:

$$\mathbf{n_0}^{\,est} = (\mathbf{R_0}^{T}\cdot \mathbf{R_0})^{-1}\cdot \mathbf{R_0}^{T}\cdot\mathbf{T_0} \quad (17)$$

The estimation of the matrix of errors is defined by formula:

$$\mathbf{d_0}^{est} = \mathbf{T_0} - \mathbf{R_0}\cdot\mathbf{n_0}^{\,est} \quad (18)$$

In assumption that the reference magnetometer has smaller errors than the tested one the differential signals are ascribed to the tested magnetometer.

These estimations coincide with real values quite well if the determined part of an additive error (temporal drift under the exponential low) is insignificant. At that it is possible to define confidential intervals of the finding estimations, using well-known statistic methods.
The confidential interval for the variance of the matrix of errors estimation:

$$N\cdot D((\mathbf{d}_0^{est})^i)/(\chi^2_{1-\alpha/2}(N-4)) < \sigma^2 < N\cdot D((\mathbf{d}_0^{est})^i)/(\chi^2_{\alpha/2}(N-4)) \qquad i=1\ldots3 \quad (19)$$

where $D((\mathbf{d}_0^{est})^i)$ – the variance of the i-th row of the estimation of the matrix of errors;
$\chi^2_{1-\alpha/2}(N-4)$, $\chi^2_{\alpha/2}(N-4)$ – fractiles of the χ^2 distribution for the significance level α.

The confidential interval for the variance of the transformation matrix estimation:

$$n^{est}_{ji} \pm t_{1-\alpha/2}(N-4)\cdot(r_{jj}\cdot D((\mathbf{d}_0^{est})^i)\cdot N/(N-4))^{0.5} \qquad i,j = 1\ldots3 \tag{20}$$

where $t_{1-\alpha/2}(N-4)$ – fractile of the Student's distribution for the significance level α;
r_{jj} – diagonal element of the matrix $(\mathbf{R_0}^T\cdot \mathbf{R_0})^{-1}$.

Otherwise the determined part could cause a great deviation of finding estimations from real values. In order to eliminate this effect we propose to separate random and determined parts of additive errors. For this it's necessary to approximate differential signals by formula, which describes accepted process for determined part. After this initial data of the test magnetometer are corrected in accordance with approximated values. Further it's necessary to repeat calculations by formula (17), (18). This technique gives considerable rise of the method tolerance, if the supposition about additive error features is true.

For description a mutual location of the sensors components and a ratio of their sensitivity we use 14 parameters – 9 angles and 5 dimensionless factors. Each sensor taken separately is described by five parameters. Nine elements of the matrix **n** are values of certain functions of these fourteen parameters. Thus we have system of nine equations with fourteen unknown terms. However, if five parameters of one of the magnetometers are defined in advance, we could determine other nine parameters, solving the set of equations by numerical methods.

Conclusions

Magnetometer data with different orientation of their sensors can be transformed to the linear model with three independent variables. Using the linear regression method and separating random and determined parts of magnetometer errors, it is possible:

a) to find estimations of the transformation matrix and the matrix of errors;
b) to find confidential intervals of these estimations;
c) to determine the set of parameters of the one of magnetometers, if the other magnetometer parameters are known.

Reference

1. Mathematical Handbook for scientists and engineers. Definitions, theorems and formulas for reference and review. G. A. Korn, T. M. Korn, McGraw-Hill Book Company, New York, 1968, pp. 556 - 558.